Development of a Computational Model for Ancient Western Warfare

Jeremy Fallick
Montgomery Blair High School

With the assistance of :

Professor Daniel Robinson
Oxford University

Professor Ashok Agrawala
University of Maryland

Abstract

The goal of this project was to create a computer model that takes information about an ancient Western battle as input and returns a prediction for the outcome of that battle. Successful models for modern combat have been created, but this project differs both in that it deals with ancient instead of modern warfare and in that it requires no user interaction, while current models do. The model was written in the Java programming language and presents a graphical user interface in which the user sets up the battle to be simulated. The battlefield is conceived as a 10 by 10 grid of tiles, each of which can contain a terrain feature and a unit. The characteristics of each unit, as well as terrain and military style, determine the results of engagements between two units. In testing, the model correctly predicted the victor in 7 out of 7 battles (100%) used for testing rather than development and 30 out of 33 battles (90.9%) overall, exceeding the goal of 80% accuracy. These results suggest that the model created can be used as a predictive tool by historians.

Introduction

The goal of this project was to provide and test an accurate and general model for ancient Western battles. The model takes information about the beginning of an ancient battle as input and returns a prediction of the victor. For the purposes of this project, "ancient warfare" comprises battles fought between c. 500 BCE and c. 500 CE. It is impossible for the scope of the model to be extended earlier than the fifth century BCE, since systematic historical accounts, required for projects of this kind, were not originated until Herodotus' work on the Persian Wars of the early fifth century [1:19]. After the fall of the Western Roman Empire in 476 CE, warfare began to change dramatically with the increasing dominance of heavily armored horsemen, culminating in the Middle Ages. Therefore, it is fitting for the scope of the model to end there.

For a combat model to be accurate, it must take into account interactions between combatants over the course of time, rather than just compare absolute strengths [2]. Models for modern combat, such as the JANUS model, have been created that conform to real-life data and are mathematically verifiable [3]. The JANUS model is a simulation of modern combat, using individual soldiers as agents and requiring interaction from the user [4], which is used to train officers in the US Army. This project differs from such combat models in two ways: first, in that it deals with a different timeframe, *i.e.*, ancient warfare as opposed to modern warfare, and second, in that it requires no user interaction; instead, units involved in the battle act according to a set of rules. Data drawn from real-world historical battles were used to develop and test the model. The target successful prediction rate was arbitrarily set at 80%.

Using the battles cited in the Results section, the model exceeded the goal of 80% accuracy in predicting the results of actual battles, suggesting that its predictions for hypothetical battles would probably reflect how they would have resulted if actually fought. Using this model, historians can predict how two armies that never fought would fare if they were opposed in battle or how a battle would have ended had some factor been different in the beginning, which would aid observers in the analysis of the importance of individual factors in the outcome. The model's predictions can also be used as evidence in evaluating the reliability of a historical account of a battle. This sort of analysis and criticism is an invaluable part of the historian's work [1:237].

Materials and Methods

Program Structure

The model was written in the Java programming language using the Eclipse software development environment and comprises six classes: Driver, Battle, Tile, Unit, Engagement, and Filter. The Driver class creates the window in which the model runs and instantiates the Battle class, which extends `javax.swing.JPanel`, a graphical user interface (GUI) component that can hold other components, in that window. The Battle class creates the GUI using the `javax.swing` package, which provides many GUI components like buttons and menus; handles user interaction by implementing `java.awt.event.ActionListener`, which handles button presses and changes to the status of the GUI made by the user, and `java.awt.event.MouseListener`, which handles mouse actions; and runs the battle simulation as a thread by implementing `java.lang.Runnable`, which allows thread creation without a subclass of Thread. The field on which the battle is fought is conceived as a 10 by 10 grid of instances of the Tile class, which extends `javax.swing.JPanel`. The Battle class contains two grids, one of which represents the current battlefield, the other of which represents the grid on the next iteration of the battle loop. The Tile class contains variables for the battlefield terrain type (grassy/temperate, desert, or snow), terrain object (empty, hill, river, woods, or impassable), the direction of the river if applicable (horizontal, top-left to bottom-right, vertical, or top-right to bottom-left), the Tile's row and column in the battlefield grid, and an instance of the Unit class. It also contains a method to clone the Tile and a paint method which graphically depicts the contents of the Tile. The Unit class contains variables to store information about the unit (Table 1). In addition, it has

Table 1: Unit Variables

Variable	**Possible Values**
Initial size	Number of men
Current size	Number of men
Unit type	Mêlée infantry, missile infantry, mêlée cavalry, or missile cavalry
Weapon	Sword (stabbing), spear (piercing), or axe/club (blunt)
Ammunition	Number of volleys
Armor	None, light, medium, or heavy
Discipline	Levy, professional, or elite
Wellness	Fresh/healthy, tired, or exhausted/sick
Facing	North, northeast, east, southeast, south, southwest, west, or northwest
Army	Attacker or defender
Row	Row number
Column	Column number
Movement turn counter	Number of turns
Missile firing turn counter	Number of turns
Battle-caused fatigue turn counter	Number of turns
Ambush target	Instance of `Unit` (or `null` if no target)
Locations containing unit	Instance of `java.util.ArrayList`

methods to draw a graphical representation of its state, to apply the changes of the current turn to its state, to turn, and to remove itself from any grids that contain it (which is called whenever the unit would rout from the field, an event determined by the unit's discipline and the proportion of the unit's initial size that remains), as well as methods that return whether it can move or fire. The Engagement class contains static methods to evaluate the results of mêlée and missile engagements. The Filter class extends `javax.swing.filechooser.FileFilter` and serves as the file filter for the model's save/load functionality.

GUI

Before running the battle model, the user must first load a saved battle or set up the battle to be simulated using the GUI. The window displayed by the program contains, from top to bottom, a grid of tiles representing the battlefield, two drop-down menus for setting military styles of the belligerents and two radio buttons to set the action performed by clicking a tile, a tabbed pane for defining what is to be inserted, and a drop-down menu for terrain type and buttons to run the simulation, clear the field, save the current battle set-up to a file of extension `.obm`, load a saved battle, and run all the battles in a folder, outputting the results to a text file in that folder (Figures 1 and A1). Units, terrain features, and ambushes can be added and removed by selecting the corresponding tab in the tabbed pane, setting the values in that tab, selecting either the “Add” or “Remove” radio button, and clicking the appropriate tile in the grid. Once the user presses the “Run” button at the bottom, the simulation begins.

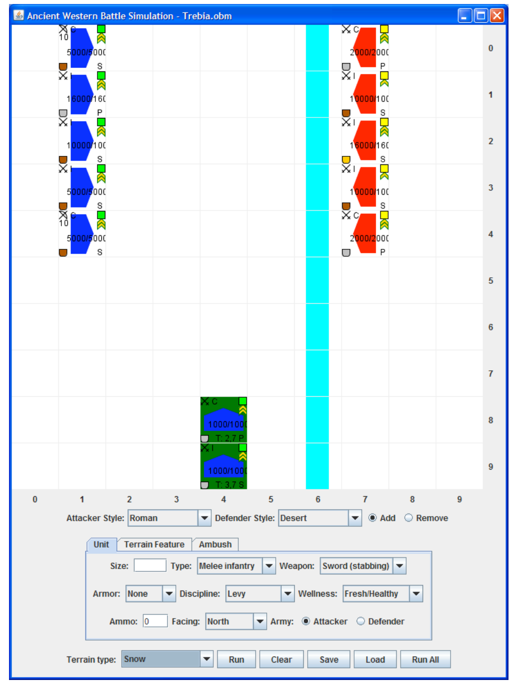

Figure 1: Screenshot of the model set up with the Battle of the Trebia

Tile Display

In every tile, information about that tile’s terrain and unit are displayed, as well as the overall battlefield terrain (Figures 2 and A2). The battlefield has three possible background colors: light green represents grassy/temperate, white represents snow, and dark yellow represents desert. If the tile contains a terrain feature, it is drawn on the tile. A river is represented by a light blue rectangle spanning the tile, woods are represented by a forest green tile background, a hill is represented by a black

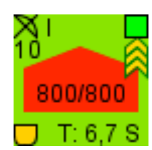

Figure 2: Screenshot of a tile containing a unit

circle inscribed in the tile, and impassable terrain is represented by a dark grey X across the tile. If the tile contains a unit, it is depicted as a pentagon pointed in the direction of the unit's facing and drawn over the terrain. The pentagon is red if the unit is of the attacking army and blue if the unit is of the defending army. The unit's current and initial sizes, respectively, are displayed on the pentagon separated by a slash. Crossed swords in the top left corner of the tile designate a mêlée unit and a bow-and-arrow designates a missile unit. Immediately to the right of this, an "I" designates infantry and a "C" designates cavalry. If the unit is a missile unit, the number of volleys of ammunition remaining to the unit is displayed below the bow-and-arrow. The color of the shield in the bottom left corner gives the unit's armor: white is none, bronze is light, silver is medium, and gold is heavy. In the bottom right corner, an "S" designates a stabbing weapon, a "P" designates a piercing weapon, and a "B" designates a blunt weapon. The color of the square in the top right corner gives the unit's wellness: green is fresh/healthy, yellow is tired, and red is exhausted/sick. The number of chevrons below the square gives the unit's discipline: one is levy, two is professional, and three is elite. Finally, if the unit has a target to ambush, the target position is displayed in the center of the bottom of the tile in the format T: [row], [column].

Simulation

When the Run button is pressed, a thread is started that runs the battle simulation. First, if the battle is being fought in the desert, every unit with medium or heavy armor becomes less well by one level. The thread then enters a loop that runs until the battle is over. An iteration of this loop is referred to as a turn. Each turn begins by setting the next-turn grid to a copy of the current grid. The thread then iterates through the tiles of the current grid. If the tile contains a unit, that unit takes an action determined by the algorithm given in Appendix B. The resulting action will be movement, the firing of a volley of missiles, or nothing. If a unit is approaching a tile, it will go around any tile into which it cannot move. No unit can move out of the grid, into impassable terrain, or into a tile containing an ally. Ambush units avoid tiles containing non-ambush units, missile units avoid tiles containing any unit, and defensive mêlée cavalry avoid tiles containing attacking mêlée units. A mêlée engagement occurs when a unit enters an occupied tile, and ends with one or both of the units being removed from the field; a missile engagement occurs when a missile unit fires on another unit, and results in the target unit losing men. Engagement evaluation is described in Appendix C. Units tire after climbing a hill, fording a river, or fighting a mêlée engagement, and recover 12 turns later. Heavy or medium infantry move every 4 turns, and heavy or medium cavalry move every 2 turns. Light and unarmored units take one fewer turn to move than their heavier counterparts, and units take one more turn to move for each level of wellness they are removed from Fresh/Healthy. Missile units can fire every other turn. The battle ends when all of one army's units are removed from the field, at which point the user is informed of the result.

Development and Testing

For the purpose of developing and testing of the model, 33 battles were researched (26 for development, 7 for testing). A list of these, as well as their sources of information, can be found in Appendix D. Save files were created for these battles. The original weights of unit parameters in engagements were created based on estimation by the author, and were then refined to fit as many of the development battles as possible. After development was complete, the model was run using the testing battles to demonstrate its performance on battles not used in development.

Results

Twenty-six battles were used to develop the model, and seven battles were used for the purpose of testing the final model alone. Because the model was developed to fit the former group, the latter group was necessary to demonstrate the model's success with battles that it was not developed to fit. The model successfully predicted the outcomes of 23 of the 26 development battles and seven of the seven testing battles (Table 2). The model was successful in predicting the outcomes of approximately 90.9% of overall battles, and 100% of the testing battles.

Table 2: Battle Results

A victory by the attacker is designated by a 0, and a victory by the defender is designated by a 1.

Battle	Actual Winner	Predicted Winner
Development		
Adrianople	0	0
Adys	0	0
Argentoratum	1	1
Beneventum 214 BCE	0	0
Cannae	1	0
Caralis	0	0
Carrhae	0	0
Castulo	1	1
Chaeronea	0	0
Cynoscephalae	0	0
Dertosa	0	0
Granicus	0	0
Heraclea	1	1
Herdonia 212 BCE	0	1
Ilipa	0	0
Lake Trasimene	0	0
Leuctra	0	0
Magnesia	0	0
Marathon	0	0
Metaurus	0	0
Pydna	1	1
Sabis	1	1
Trebia	1	1
Tunis	0	0
Watling Street	1	0
Zama	0	0
Testing		
Delium	1	1
Amphipolis	0	0
Mantinea 418 BCE	1	1
Nemea	0	0
Gaza 312 BCE	0	0
Raphia	0	0
Sellasia	0	0

Discussion

The target for the model's success rate was 80%. The final version of the model exceeded this goal both for testing battles (100%) and overall (90.9%). The question of how valid these results are in judging the accuracy of the model, however, remains.

Statistical Analysis

A Z-test using the observed success rate of 90.9% with a null hypothesis that the actual success rate is 80% and an alternate hypothesis that the actual success rate is less than 80% gives a p-value of 0.9414. In other words, with an observed success rate as high as 90.9%, there is a 5.86% chance that the model's actual success rate is in fact less than the target 80%. Because this is so low, it suggests that the model has met the 80% goal. However, this test and the conclusions drawn from it may not be valid. The battles used for the model were not selected randomly; they were chosen by the author based on availability of information. While this was necessary because only battles with enough information could be used, it is possible that there is bias as a result of this, not only in the analysis of the results but also in the development of the model itself. This bias would be towards battles involving Greek or Roman combatants, because it is from Greek and Roman historians that we get first-hand accounts of battles. In addition, the accuracy of current information about ancient battles is not entirely reliable, so it is possible that the battles used for this project were not accurately reproduced.

Possibilities for Future Research

This project has resulted in the creation of a predictive tool for historians. However, this model does have the potential for improvement. The most obvious way to improve the model and ensure its accuracy is to research more battles and to incorporate these into the model. As historical knowledge is uncertain and can change with new discoveries, more accurate information about the battles already used can surface. Incorporating this new information into the model would also help to improve its accuracy. Improvement is also possible for the battle engine itself. Another set of rules governing unit action than that used in this project might have greater success predicting battles, and be more representative of the usual course of battle.

Appendix A: Labeled Screenshots

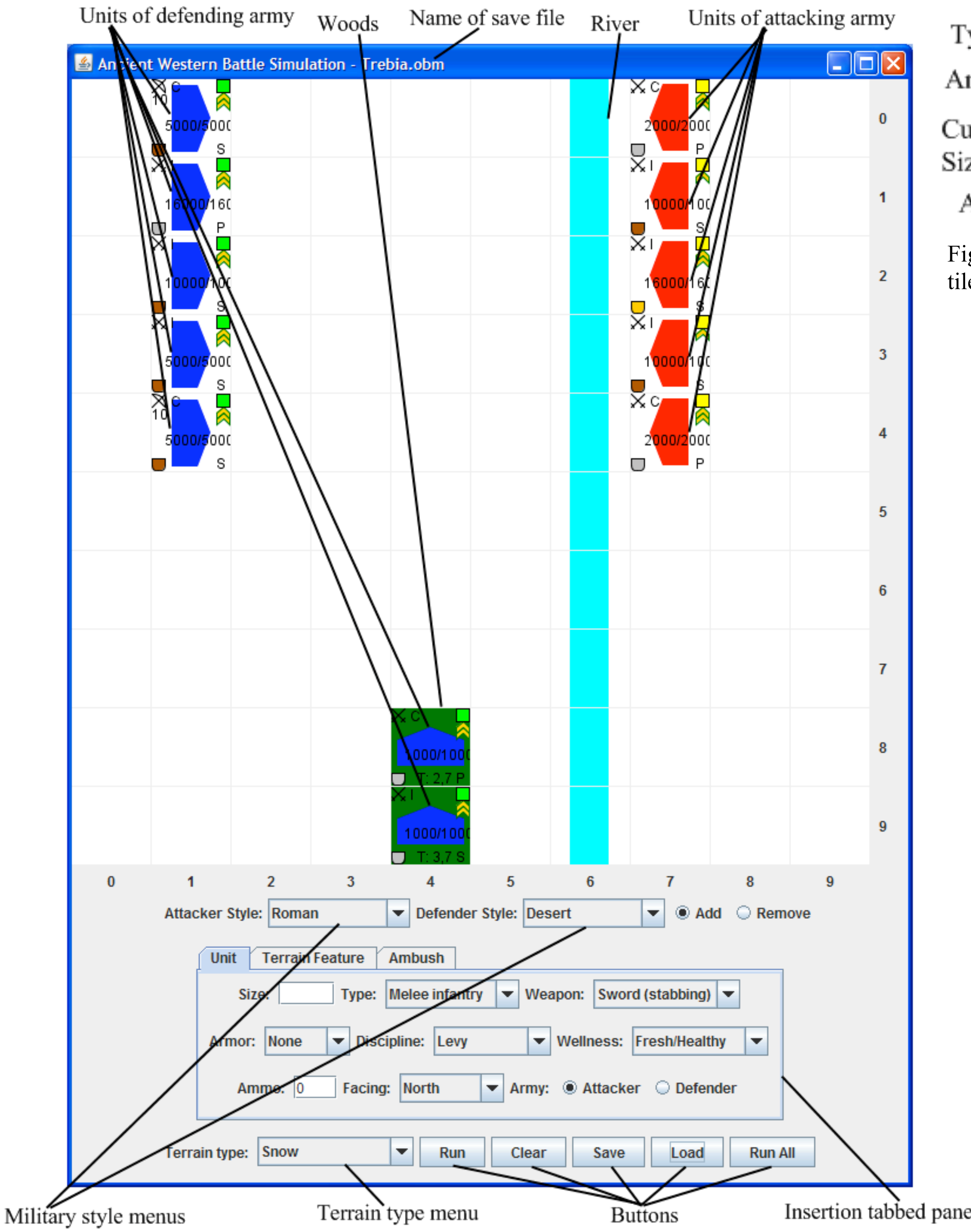

Figure A1: Screenshot of the model set up with the Battle of the Trebia (labeled)

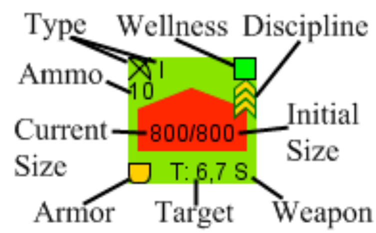

Figure A2: Screenshot of a tile containing a unit (labeled)

Appendix B: Unit Action Determination

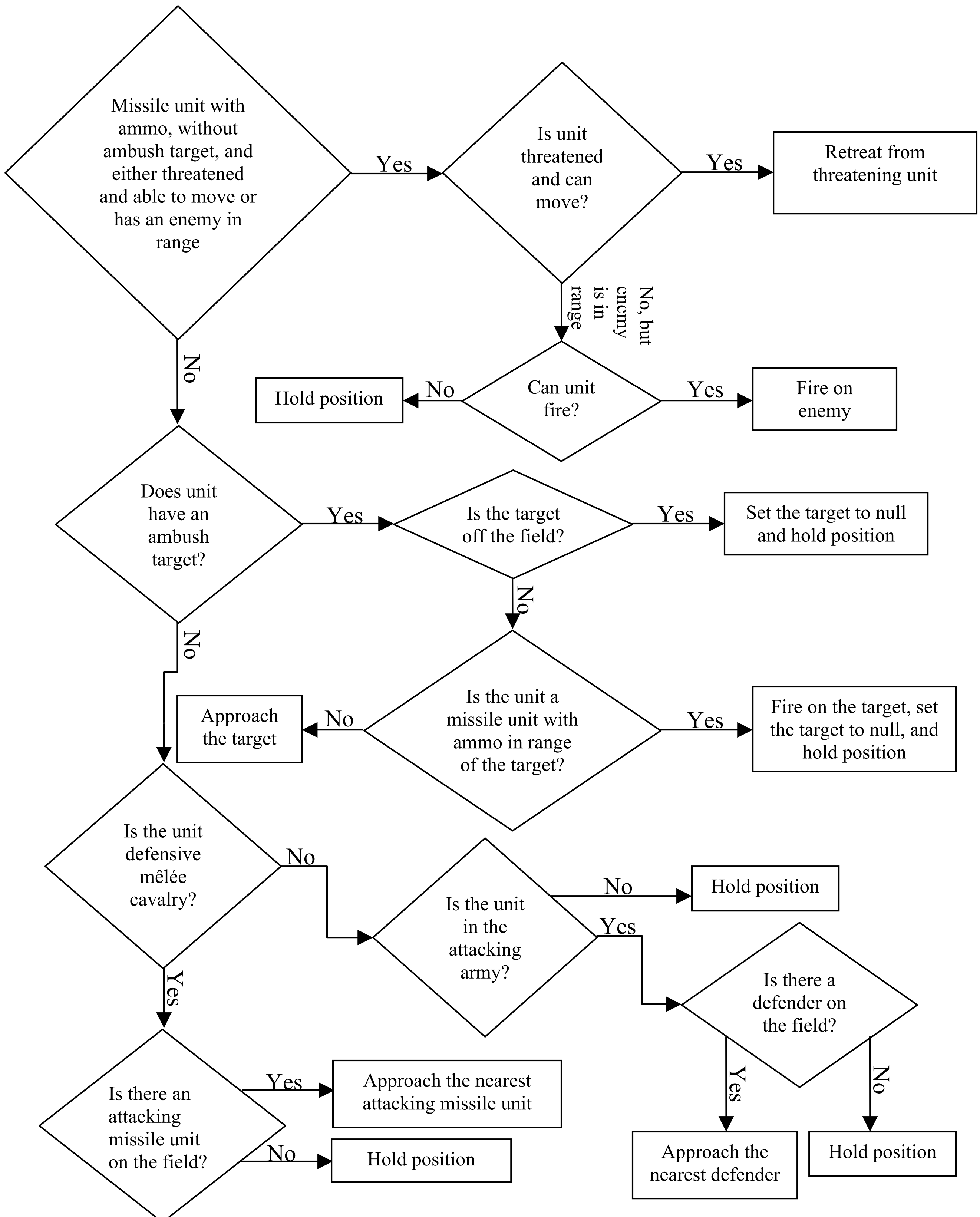

Figure A3: Unit Action Selection Algorithm Flowchart

Appendix C: Engagement Evaluation

Mêlée Engagement

The unit considered the attacker is the unit entering the occupied tile, and the tile's occupier is considered the defender. First, a base count of the casualties the attacker can inflict upon the defender is found. This value is initially set to the size of the attacker. It is then multiplied by the following modifiers:

1.1 + max((attacker's armor (unarmored is 0, heavy is 3) * 0.5 if defender has blunt weapon, 1 otherwise - defender's armor * 0.5 if defender has blunt weapon, 1 otherwise), -2.5) * 0.4

1.1 + max((attacker's discipline (levy is 1, elite is 3) – defender's discipline), -2.5) * 0.4;

1 + (defender's wellness (fresh/healthy is 0, exhausted/sick is 2) – attacker's wellness) * 0.3

0.9 if attacker has a piercing weapon

1.1 if attacker has a blunt weapon

3 if attacker has a piercing weapon and defender is cavalry

0.4 if tile of engagement contains a hill

0.8 if attacker is cavalry and tile contains woods

2 if defender is being attacked in the rear

1.5 if defender is being attacked in the flank

2 if attacker is cavalry

0.75 if the attacker is a missile unit

3 if defender is attacker's ambush target

1.1 if attacker's army has Roman military style and attacker is medium or heavy mêlée infantry with a stabbing weapon

1.1 if attacker's army has Hellenic military style and attacker is mêlée infantry with a piercing weapon

1.1 if attacker's army has Hellenistic military style and attacker is medium or heavy mêlée cavalry with a piercing weapon

1.1 if attacker's army has Desert military style and attacker is unarmored or light mêlée infantry

1.1 if attacker's army has Celto-Germanic military style and attacker is infantry

1.1 if attacker's army has Steppe Nomadic military style and attacker is missile cavalry

The count of the casualties that the defender can inflict is found using the same armor, discipline, and wellness modifiers. It is then multiplied by the following modifiers:

1.1 if defender has a piercing weapon

9 if defender has a blunt weapon

3 if defender has a piercing weapon and attacker is cavalry

0.9 if attacker is cavalry and tile contains woods

0.2 if defender is being attacked in the rear

0.4 if defender is being attacked in the flank

0.75 if the defender is cavalry

0.75 if the defender is a missile unit

0.3 if the defender is the attacker's ambush target

The same style modifiers apply, except the Celto-Germanic modifier is removed, and the Hellenistic modifier applies to mêlée infantry instead of cavalry.

Next, the number of casualties each unit can sustain before routing is determined with the following formula:

Current size – initial size * (4 – discipline) * 0.06

The winner is determined by comparing the ratio between each unit's sustainable casualty count and the count of casualties its opponent can inflict. The unit for which this ratio is greater wins the engagement; if the ratios are equal the engagement ends in a draw. The loser is then removed from the field and the winner takes casualties equal to the winner's sustainable casualty count times the count of casualties the loser can inflict divided by the winner's pre-engagement size and is placed in the contested tile. In the event of a draw, both units are removed from the field.

Missile Engagement

At the beginning of a missile engagement, the aggressor turns to face its target and its ammunition is decreased by one volley. The probability of each missile hitting is calculated by multiplying 0.1 + 0.025 * (aggressor's discipline – 1) by the following modifiers:

1.25 if the aggressor is on a hill

0.5 if the aggressor is in woods

0.5 if the target is in woods

0.5 * aggressor's discipline + 0.5

The probability of each hit being fatal is calculated by multiplying 0.5 by the following modifiers:

1 - 0.2 * target's armor if the target is not unarmored

1.25 if the target is being shot in the rear

1.125 if the target is being shot in the flank

Finally, the number of kills is calculated by multiplying the size of the aggressor by the probability of hitting and the probability of killing. This number of casualties is inflicted on the target.

Appendix D: Battle Sources

Development:
Battle of the Trebia [S1] [S2]
Battle of the Granicus [S1] [S3]
Battle of Marathon [S1] [S4]
Battle of Leuctra [S1] [S5]
Battle of Chaeronea [S1] [S6]
Battle of Cannae [S1] [S7]
Battle of Zama [S1] [S8]
Battle of the Sabis (Sambre) [S1] [S9]
Battle of Adrianople [S1] [S10]
Battle of Lake Trasimene [S1] [S11]
Battle of Heraclea [S12]
Battle of Adys [S13] [S14]
Battle of Tunis [S13] [S15]
Battle of Dertosa (Ibera) [S16]
Battle of Caralis (Cornus) [S17]
Battle of Beneventum (214 BCE) [S18]
Battle of Herdonia (212 BCE) [S19]
Battle of Castulo [S20]
Battle of the Metaurus [S21]
Battle of Ilipa [S22]
Battle of Cynoscephalae [S23] [S24]
Battle of Pydna [S25] [S26]
Battle of Magnesia [S27] [S28]
Battle of Watling Street [S29]
Battle of Carrhae [S30] [S31]
Battle of Argetoratum [S32]
Testing:
Battle of Delium [S33]
Battle of Amphipolis [S34]
Battle of Mantinea (418 BCE) [S35] [S36]
Battle of Nemea [S37]
Battle of Gaza (312 BCE) [S38]
Battle of Raphia [S39]
Battle of Sellasia [S40]

Sources:
[S1] The History Department at the United States Military Academy, *Atlas for Ancient Warfare*. [Online]. Available: http://www.dean.usma.edu/history/web03/atlases/AncientWarfare/. [Accessed: July 20, 2009].

[S2] J. Rickard, "Battle of the Trebia, late December 218 BC," *Military History Encyclopedia on the Web*. [Online]. Available: http://www.historyofwar.org/articles/battles_trebia.html. [Accessed: July 20, 2009].

[S3] M. Moerbeek, "The battle of Granicus, 334 BC," *HELLAS:NET*, January 21, 1998. [Online]. Available: http://monolith.dnsalias.org/~marsares/warfare/battle/granicus.html. [Accessed: July 21, 2009].

[S4] J. Lendering, "Battle of Marathon," *Livius,* August 11, 2007. [Online]. Available: http://www.livius.org/man-md/marathon/marathon.html. [Accessed: July 21, 2009].

[S5] "Battle of Leuctra," *Encyclopædia Britannica Online,* 2009. [Online]. Available: http://www.britannica.com/EBchecked/topic/337705/Battle-of-Leuctra. [Accessed: July 21, 2009].

[S6] Diodorus of Sicily, *Bibliotheca historica (Library of History)*, M. Crawford and D. Whitehead, Trans., 16.85.5-86.

[S7] A. von Schlieffen, *Cannae*, Ch. 1. No more information given.

[S8] "Battle of Zama," *UNRV History*. [Online]. Available: http://www.unrv.com/empire/battle-of-zama.php. [Accessed July 23, 2009].

[S9] J. Rickard, "Battle of the Sambre, July 57 B.C.," *Military History Encyclopedia on the Web*. [Online]. Available: http://www.historyofwar.org/articles/battles_sambre_57_bc.html. [Accessed: July 23, 2009].

[S10] D. Koeller, "Battle of Adrianople: 378," *WebChron: The Web Chronology Project*, 1999. [Online]. Available: http://www.thenagain.info/WebChron/Mediterranean/Adrianople.html. [Accessed: July 23, 2009].

[S11] M. Burk, "Battle of Lake Trasimene, Italy 217 B.C.E," *Suite101.com*, December 8, 2008. [Online]. Available: http://ancient-military-history.suite101.com/article.cfm/battle_of_lake_trasimene_italy_217_bce. [Accessed: July 24, 2009].

[S12] J. Jonas, "The Initial Clash: Republican Rome vs. Pyrrhus of Epirus," *AncientBattles.com*, January 20, 2007. [Online]. Available: http://www.ancientbattles.com/WAB_Successors/EpiroteScenario1.htm. [Accessed: July 24, 2009].

[S13] J. Rickard, "First Punic War, 264-241 BC," *Military History Encyclopedia on the Web*. [Online]. Available: http://www.historyofwar.org/articles/wars_punic1.html. [Accessed: July 24, 2009].

[S14] J. Rickard, "Battle of Adys, 256 BC," *Military History Encyclopedia on the Web*. [Online]. Available: http://www.historyofwar.org/articles/battles_adys.html. [Accessed: July 24, 2009].

[S15] J. Rickard, "Battle of Tunis, 255 BC," *Military History Encyclopedia on the Web*. [Online]. Available: http://www.historyofwar.org/articles/battles_tunis.html. [Accessed: July 24, 2009].

[S16] D. Fournie, "The Battle of Dertosa, 215 B.C.," *The Great Battles of History Series*, 1994. [Online]. Available: http://www.gmtgames.com/c3i/1_4_dertosa.pdf. [Accessed: July 27, 2009].

[S17] D. Fournie, "Caralis: The Battle for Sardinia, 215 B.C.," *The Great Battles of History Series*. [Online]. Available: http://www.gmtgames.com/c3i/1_9_Caralis%20The%20Battle%20for%20Sardinia.pdf. [Accessed: July 27, 2009].

[S18] T. Livius, *Ab Urbe Condita (History of Rome)*, C. Roberts, Trans., 24.11, 14-16

[S19] T. Dodge, *Hannibal: A history of the art of war among the Carthaginians and Romans down to the Battle of Pydna, 168 B.C., with a detailed account of the Second Punic War*. Adamant Media Corporation, 2001, pp. 496-497

[S20] D. Fournie, "The Battle of Castulo, 211 B.C.," *The Great Battles of History Series*. [Online]. Available: http://www.gmtgames.com/c3i/1_5_Battle_Castulo.pdf. [Accessed: July 28, 2009].

[S21] C. Jones, "Battle of the Metaurus (207 BC)," *Fanaticus*, January 31, 2000. [Online]. Available: http://www.fanaticus.org/DBA/battles/metaurus.html. [Accessed July 28, 2009].

[S22] J. Lazenby, *Hannibal's War: a military history of the Second Punic War*. University of Oklahoma Press, 1998, pp. 145-150.

[S23] N. Markoulakis, "The Battle of Cynoscephalae," *Σparta*, October 2007. [Online]. Available: http://www.sparta.markoulakispublications.org.uk/index.php?id=138. [Accessed July 29, 2009].

[S24] Polybius of Megalopolis, *The Histories*, W. Paton, Trans., 18.23-26.

[S25] S. Skarmintzos, "The Battle of Pydna 168 BC," *Fanaticus*, October 4, 2005. [Online]. Available: http://www.fanaticus.org/DBA/battles/Pydna/index.html. [Accessed July 29, 2009].

[S26] J. Foss, "The Third Macedonian War and the Battle of Pydna," *The English Times of Katerini*, January 2001. [Online]. Available: http://www.musesnet.gr/etimes/BATTLE OF PYDNA.htm. [Accessed July 29, 2009].

[S27] J. Rickard, "Battle of Magnesia, December 190 B.C.," *Military History Encyclopedia on the Web*. [Online]. Available: http://www.historyofwar.org/articles/battles_magnesia.html. [Accessed: July 29, 2009].

[S28] Appian of Alexandria, *Ῥωμαϊκά (Roman History)*, H. White, Trans., Vol. 11, §31-32.

[S29] C. Brantley, "Battle of Watling Street (61 AD)," *Fanaticus*, June 21, 2002. [Online]. Available: http://www.fanaticus.org/DBA/battles/watlingstreet61AD.html/. [Accessed July 30, 2009].

[S30] C. Heaton, "Crassus and Parthia," *United Nations of Roma Victrix*. [Online]. Available: http://www.unrv.com/fall-republic/crassus-and-parthia.php. [Accessed July 30, 2009].

[S31] C. Heaton, "Battle of Carrhae," *United Nations of Roma Victrix*. [Online]. Available: http://www.unrv.com/fall-republic/battle-of-carrhae.php. [Accessed July 30, 2009].

[S32] C. Jones, "Battle of Argentoratum (Strasbourg) (357 AD)," *Fanaticus*, January 17, 2002. [Online]. Available: http://www.fanaticus.org/DBA/battles/argentoratum.html. [Accessed July 30, 2009].

[S33] C. Brantley, "The Battle of Delium (424 BC)," *Fanaticus*, June 9, 2007. [Online]. Available: http://www.fanaticus.org/DBA/battles/Delium/index.html. [Accessed August 5, 2009].

[S34] J. Bury, *History of Greece to the death of Alexander the Great*, 2nd ed. Macmillan, 1922, pp. 453-454.

[S35] L. Ueda-Sarson, "The First Battle of Mantinea: 418 BC," *Luke Ueda-Sarson's Historical Battle Scenarios for DBM*, October 6, 2000. [Online]. Available: http://www.ne.jp/asahi/luke/ueda-sarson/Mantinea1.html. [Accessed August 5, 2009].

[S36] Thucydides, *History of the Peloponnesian War*, R. Crawley, Trans., 5.67.

[S37] J. Jones, "Ancient Battles War Gaming Introduction," *AncientBattles.com*, May 8, 2008. [Online]. Available: http://www.ancientbattles.com/AtG_GreekMercs.htm. [Accessed: August 5, 2009].

[S38] K. Barwise, "Battle of Gaza (312 BC)," *Fanaticus*, July 3, 1999. [Online]. Available: http://www.fanaticus.org/DBA/battles/gaza312bc.html. [Accessed: August 6, 2009].

[S39] Polybius of Megalopolis, *The Histories*, W. Paton, Trans., 5.65, 79, 82, 84-85.

[S40] Polybius of Megalopolis, *The Histories*, W. Paton, Trans., 2.65-69.

References

[1] R. G. Collingwood, J. van der Dussen, Ed., *The Idea of History*. New York: Oxford University Press, 1994.

[2] H. Friman and B. Brehmer, "Using Microworlds to Study Intuitive Battle Dynamics: A concept for the future," presented at Command & Control Research & Technology Symposium, U.S. Naval War College, Newport, Rhode Island, United States of America, 1996.

[3] L. Ingber, H. Fujio, and M. F. Wehner, “Mathematical comparison of combat models to exercise data,” *Mathematical Computer Modelling*, Vol. 15, No. 1, pp. 65-90, 1991.

[4] United States Army, “JANUS,” *Army Modeling and Simulation Resource Repository*, May 10, 2006. [Online]. Available: www.msrr.army.mil. [Accessed: May 17, 2009]